The third midterm covers Sections 14.7 and 15.1–15.7 of the textbook, except for the topics related to probability (pages 985–987).

The practice exam problems below are pretty representative of what you can expect for the midterm in terms of the difficulty level, nature of problems, and length of the exam, but not in terms of specific questions or topics covered. The latter is due to the simple statistical fact that the topics that can be covered on any given exam represent only a small random snapshot of the entire exam material, and another such random snapshot is likely to result in different questions and topics. Thus, simply studying the problems below will not adequately prepare you. The only way to be fully prepared for the exam is to work through the entire exam material, including all assigned homework.

Practice Exam Problems:

1. Find all critical points of the function

   \[ f(x, y) = x^2 + y^2 + x^2 y \]

   and classify them as local maxima, local minima, or saddle points.

2. Evaluate the integral \( \int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} \, dx \, dy \). (Hint: Sketch the region and reverse the order of integration.)

3. Evaluate the integral \( \int \int_R xy \, dA \), where \( R \) is the region in the first quadrant that lies between the circles \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 4 \) (i.e., a quarter annulus).

4. Find the volume of the region that lies inside the cylinder \( x^2 + y^2 = 1 \), below the plane \( z = 4 \), and above the surface \( z = 1 - x^2 - y^2 \).

5. (a) Express the point \( (x, y, z) = (1, 1, 1) \) in cylindrical coordinates.
(b) Convert the equation (in cylindrical coordinates)
\[ r = 2 \cos \theta \]
to rectangular coordinates and identify the surface represented by the equation. (Be specific, saying, for example, “a half cone, centered at the origin, whose surface forms an angle \( \frac{\pi}{6} \) with the positive z-axis”, instead of “a cone”.)

6. The following problems are independent of one another. In each case, set up, but do not evaluate, an iterated integral in the coordinate system specified. (Hint: You may want to sketch the appropriate regions to determine the correct integration limits.)

(a) Express, as a **double integral in rectangular coordinates**, the volume of a tetrahedron with corners \((0,0,0), (1,0,0), (0,2,0),\) and \((0,0,3)\).

(b) Express, as a **double integral in polar coordinates**, the volume of the “spherical cap” consisting of the portion of the solid sphere of radius 2 centered at the origin that lies above the plane \( z = 1 \).

(c) Express, as a **double integral**, the mass \( m \) of a lamina occupying the triangle with vertices \((0,0), (0,3),\) and \((1,1)\), if the density at any point inside the triangle is equal to the distance of that point from the \( x \)-axis.